1

a) $\overrightarrow{AC} = \langle 2-1, 0-0, 1-0 \rangle = \langle 1, 0, 1 \rangle$

$\overrightarrow{AB} = \langle 2-1, 1-0, 2-0 \rangle = \langle 1, 1, 2 \rangle$

b) $\overrightarrow{AC} \times \overrightarrow{AB} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 1 \\ 1 & 1 & 2 \end{vmatrix} = -\vec{i} - \vec{j} + \vec{k} = \vec{N}$

$\overrightarrow{AC} \cdot \vec{N} = +1 \cdot -1 + 0 \cdot 1 + 1 \cdot 1 = 0$

$\overrightarrow{AB} \cdot \vec{N} = 1 \cdot (-1) + 1 \cdot (-1) + 2 \cdot 1 = 0$

c) To be a point $P(x, y, z)$ in the plane then the vector $\overrightarrow{AP} = \langle x-1, y, z \rangle$ must be perpendicular to the normal vector $\vec{N}$. Thus

$\vec{N} \cdot \overrightarrow{AP} = 0 \Rightarrow (-1)(x-1) + (-1) \cdot y + 1 \cdot z = 0$

$1 - x - y + z = 0$

$1 = x + y - z$ ☆

$A(1,0,0)$ $B(2,1,2)$ $C(2,0,1)$

☆ $1 = 1 + 0 - 0 = 1$ ✓ $1 = 2 + 1 - 2 = 1$ ✓ $1 = 2 + 0 - 1 = 1$ ✓

2

a) $3\cos t = 0 \Rightarrow \boxed{t = \frac{\pi}{2}}$

$\sin t = \cos s$

$0 = 2 \sin s \Rightarrow \boxed{s = 0}$

$\Rightarrow 1 = \sin\frac{\pi}{2} = \cos 0 = 1$

point of intersection is $\langle 0, 1, 0 \rangle$

b) $\vec{r}_1'(t) = \langle -3\sin t, \cos t, 0 \rangle$

$\vec{r}_2'(s) = \langle 0, -\sin s, 2\cos s \rangle$

Find the tangent vectors to the curves at the intersection point

$\vec{r}_2'(0) \cdot \vec{r}_1'\left(\frac{\pi}{2}\right) = \langle 0, 0, 2 \rangle \cdot \langle -3, 0, 0 \rangle = 0 \implies$ angle of intersection is $\frac{\pi}{2}$

(3) a) $\vec{A} \perp \vec{A} \times \vec{B} \implies \text{proj}_{\vec{A} \times \vec{B}} \vec{A} = 0$

b) $\text{orth}_{\vec{A} \times \vec{B}} \vec{A} = \vec{A} - \text{proj}_{\vec{A} \times \vec{B}} \vec{A} = \vec{A}$

(4) $\vec{a} + \vec{b} + \vec{c} = 0$

$\vec{a} \times (\vec{a} + \vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c} = 0$

$\implies \vec{a} \times \vec{b} = -\vec{a} \times \vec{c}$

$\vec{b} \times (\vec{a} + \vec{b} + \vec{c}) = \vec{b} \times \vec{a} + \vec{b} \times c = 0$

$\implies \vec{b} \times \vec{a} = -\vec{b} \times \vec{c} \implies \vec{a} \times \vec{b} = \vec{b} \times \vec{c}$

$\implies -\vec{a} \times \vec{c} = \vec{a} \times \vec{b} = \vec{b} \times \vec{c}$ ☆

Using defn of length of the vector product

$\|\vec{a} \times \vec{b}\| = \|\vec{a}\| \|\vec{b}\| \sin\theta$ $\qquad \theta$ angle between $\vec{a}$ & $\vec{b}$

$= \|\vec{c} \times \vec{a}\| = \|\vec{a}\| \|\vec{c}\| \sin B$

$= \|\vec{a} \times \vec{b}\| = \|\vec{a}\| \|\vec{b}\| \sin C$ $\implies \|a\| \|b\| \sin C = \|\vec{c}\| \|a\| \sin B = \|\vec{b}\| \|\vec{c}\| \sin A$

$= \|\vec{b} \times \vec{c}\| = \|\vec{b}\| \|\vec{c}\| \sin A$

dividing by $\|a\| \|b\| \|c\|$

we get

$$\frac{\sin C}{\|\vec{c}\|} = \frac{\sin B}{\|\vec{b}\|} = \frac{\sin A}{\|\vec{a}\|}$$

⑦ $\vec{v}(t) = t\vec{i} + t^2\vec{j}$ $\qquad$ $\vec{a}(t) = \vec{i} + 2t\vec{j}$

$T(t) = \dfrac{v(t)}{\|v(t)\|} = \dfrac{\vec{i} + t\vec{j}}{\sqrt{1+t^2}}$ $\qquad$ $N(t) = \dfrac{T'(t)}{\|T'(t)\|} = \dfrac{-t\vec{i} + \vec{j}}{\sqrt{t^2+1}}$

$a_T = a(1) \cdot T(1) = \dfrac{3}{\sqrt{2}}$ $\qquad$ $a_N = a(1) \cdot N(1) = \dfrac{1}{\sqrt{2}}$

⑧ $\left\| \begin{vmatrix} i & j & k \\ 3 & 4 & -1 \\ 2 & -4 & 2 \end{vmatrix} \right\| = \sqrt{4^2 + 8^2 + 20^2} = ?$

(5) Let $z = f(x,y) = 2x^2 + 3y^3$

define $F(x,y,z) = 2x^2 + 3y^3 - z = 0$

to find normal vector to level surface take ∇ at $(1,1,5)$

$$\vec{\nabla} F(1,1,5) = 4(1)\vec{i} + 9(1)^2\vec{j} - \vec{k} = \langle 4, 9, -1 \rangle$$

thus the tangent plane $\quad P_0 = (1,1,5) \quad P = (x,y,z)$

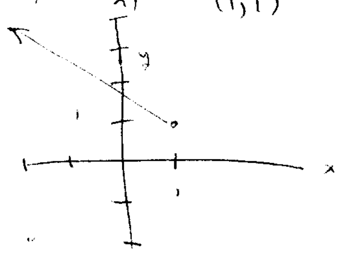
$$\vec{\nabla} F(1,1,5) \cdot \underbrace{\langle x-1, y-1, z-5 \rangle}_{\text{" } \overrightarrow{P_0P} \text{ (vector in plane)}} = 0$$

$$4(x-1) + 9(y-1) - (z-5) = 0$$

$$4x + 9y - z = 8$$

(6)

a) $\vec{\nabla} f$ at $(1,1)$ is $-3\vec{i} + 3\vec{j}$



b) $D_{\vec{u}} f = \vec{\nabla} f \cdot \vec{u} = \| \vec{\nabla} f \| \, \| \vec{u} \| \cos\theta \quad$ θ angle between $\vec{\nabla} f$ & $\vec{u}$

so we want $\theta = 0$

thus maximal directional derivative is $\| \vec{\nabla} f \| = \sqrt{18} = 3\sqrt{2}$

c) $\vec{u}$ must be in the same direction as $\vec{\nabla} f$ and with norm 1

$$\vec{u} = \frac{\vec{\nabla} f}{\| \vec{\nabla} f \|} = \frac{-1}{\sqrt{2}}\vec{i} + \frac{1}{\sqrt{2}}\vec{j}$$