Practice Midterm

(1) The three points $A(1,0,0)$ $B(2,1,2)$ and $C(2,0,1)$ lie in a plane

a) Determine two vectors which lie in the plane. Clearly indicate how you constructed these vectors.

b) Hence determine a normal vector to the plane. Check your answer

c) Finally, determine the equation of the plane. Check your answer

d) What is the shortest distance from this plane to the point $D(3,1,8)$

(2) Consider the following two curves given in parametric form:

C_1 : $\vec{r}_1(t) = \langle 3\cos t, \sin t, 0 \rangle$ $\quad 0 \le t < \pi$

C_2 : $\vec{r}_2(t) = \langle 0, \cos t, 2\sin t \rangle$ $\quad 0 \le t < \pi$

a) Determine the point of intersection of the two curves.

b) Determine the angle of intersection of the curves

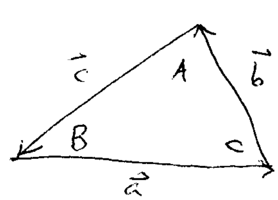
(3) If $\vec{A} \times \vec{B} \neq \vec{0}$

a) $\text{proj}_{\vec{A}\times\vec{B}} \vec{A} =$

b) $\text{orth}_{\vec{A}\times\vec{B}} \vec{A} =$

(4) Use vector methods to prove that for a triangle with sides given by the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ and opposite angles A, B, C

$$\frac{\sin A}{\|\vec{a}\|} = \frac{\sin B}{\|\vec{b}\|} = \frac{\sin C}{\|\vec{c}\|}$$



(5) Find the tangent plane to the graph of $f(x,y) = 2x^2 + 3y^3$ at the point $(1,1,5)$

(6) Assume that at the point $(1,1)$, $\frac{\partial f}{\partial x} = -3$, $\frac{\partial f}{\partial y} = 3$

a) Draw $\vec{\nabla f}$ at $(1,1)$

b) What is the maximal directional derivative of f at $(1,1)$?

c) For what $\vec{u}$ is $f'_{\vec{u}}$ at $(1,1)$ maximal? Write $\vec{u}$ in the form $x\vec{i} + y\vec{j}$.

(7) Let $\vec{r}(t) = \frac{t^2}{2}\vec{i} + \frac{t^3}{3}\vec{j}$, Find a_T & a_N for $t=1$.

(8) Find the area of the parallelogram spanned by $\vec{A} = 3\vec{i} + 4\vec{j} - \vec{k}$ and $\vec{B} = 2\vec{i} - 4\vec{j} + 2\vec{k}$

This practice midterm does not cover all the material you have learned, please look at the other practice midterm.