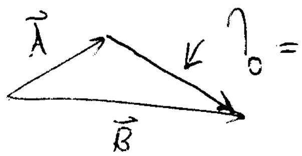
1 Let A, B, C be the vertices of a triangle. Let $\vec{A} = \overrightarrow{OA}$, $\vec{B} = \overrightarrow{OB}$, $\vec{C} = \overrightarrow{OC}$
$\vec{C} = \overrightarrow{OC}$

Let P be the point that is on the line ~~[illegible]~~ segment joining A to the mid point of the edge BC and twice as far from A as from the mid point.

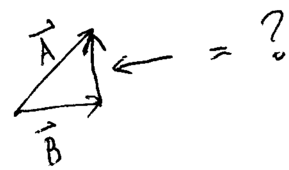
Show that $\overrightarrow{OP} = \dfrac{\vec{A} + \vec{B} + \vec{C}}{3}$

2 2

a)



b)



3 3 Let $\vec{A} = \vec{i} + \vec{j}$, $\vec{B} = \dfrac{1+\sqrt{3}}{2}\vec{i} + \dfrac{1-\sqrt{3}}{2}\vec{j}$

a) Compute $\vec{A} \cdot \vec{B}$ using Thm 3 on pg. 697

b) " " definition of dot product (angle between these 2 vectors is $\frac{\pi}{3}$)

c) Do a) & b) agree?

4 4 Let $\vec{A} = \vec{i} - 2\vec{j} - \vec{k}$

Compute

a) $\text{proj}_{\vec{j}} \vec{A}$

b) $\vec{B} = -\vec{i} + 4\vec{k} + 3\vec{j}$

$\text{proj}_{\vec{B}} \vec{A}$

5 5 $\vec{A}, \vec{B} \neq \vec{0}$

When is $\text{proj}_{\vec{B}} \vec{A} = \text{proj}_{\vec{A}} \vec{B}$?

6 6 a) Find the equation of the plane passing through $(1, 1, 3)$ & perpendicular to vector $3\vec{i} + 4\vec{j} + 5\vec{k}$

b) What is the distance from this plane to the point $(2, 8, 4)$

7 Given $\vec{A} = 2\vec{i} + 3\vec{j}$, $\vec{B} = \vec{i} - 6\vec{j}$

a) Draw the parallelogram formed by these two vectors

b) Find the area of this parallelogram

8 8 Let $\vec{A} = \vec{i} + 3\vec{j} + 4\vec{k}$, $\vec{B} = 2\vec{i} + \vec{j} + \vec{k}$

a) Compute the vector perpendicular to $\vec{A}, \vec{B}$

b) What is the area of the parallelogram formed from $\vec{A}, \vec{B}$

c) Let $\vec{C} = -\vec{i} + 3\vec{j} - 5\vec{k}$

what is the volume of the parallelipiped from $\vec{A}, \vec{B}, \vec{C}$

d) What is $A \times (B \times C)$ using the formula on pg. 727

9 9 The position of particle moving through space is given by the vector function

$\vec{G}(t) = 10 \cos t \, \vec{i} + 10 \sin t \, \vec{j}$

a) What is the tangent vector at $t = \frac{\pi}{4}$?

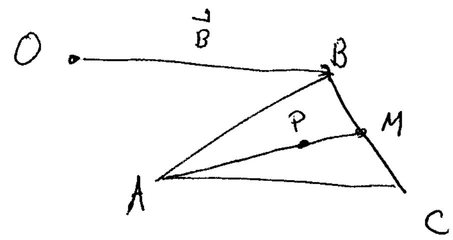
b) What is $\| G(t) \|$?

c) Draw the path of the particle

d) What $G(t) \cdot G'(t)$?

e) What is the angle between $G(t) \cdot G'(t)$?

1



M midpoint of $\overline{BC}$

$$2\,|\overline{PM}| = |\overline{AP}|$$

Length of line segment

Show $\overrightarrow{OP} = \dfrac{\vec{A} + \vec{B} + \vec{C}}{3}$

We know $2\overrightarrow{PM} = \overrightarrow{AP}$

$$2(\vec{M} - \vec{P}) = \vec{P} - \vec{A}$$

$$2\vec{M} + \vec{A} = \vec{P} + 2\vec{P} = 3\vec{P}$$

we need to find $\vec{M}$

$$\vec{M} = \vec{B} + \tfrac{1}{2}\overrightarrow{BC}$$

$$2\left(\vec{B} + \tfrac{1}{2}(\vec{C} - \vec{B})\right) + \vec{A} = 3\vec{P}$$

$$\frac{\vec{A} + \vec{B} + \vec{C}}{3} = \vec{P}$$

3

a) $\vec{A} \cdot \vec{B} = \dfrac{1+\sqrt{3}}{2} \cdot 1 + \dfrac{1-\sqrt{3}}{2} \cdot 1 = 1$

b) $\|\vec{A}\| = \sqrt{2}$, $\|\vec{B}\| = \sqrt{\left(\frac{1+\sqrt{3}}{2}\right)^2 + \left(\frac{1-\sqrt{3}}{2}\right)^2} = \frac{1}{2}\sqrt{1+2\sqrt{3}+3+1-2\sqrt{3}+3}$

$= \sqrt{2}$

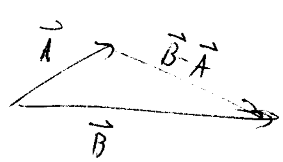
$\vec{A} \cdot \vec{B} = \|\vec{A}\|\,\|\vec{B}\| \cos\theta$ $\quad \theta = \frac{\pi}{3}$

$= 2 \cdot \frac{1}{2} = 1$

c) yes

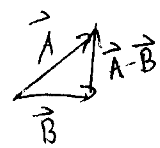
2

a)



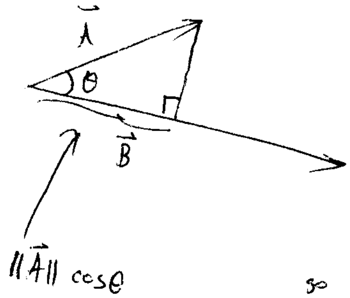
$\vec{A} + (\vec{B} - \vec{A}) = \vec{B}$

b)



(4)

$$\cos\theta = \frac{adj}{hyp}$$



so $\quad \mathrm{proj}_{\vec{B}} \vec{A} = \|\vec{A}\| \cos\theta \; \hat{u}$ $\qquad$ $\hat{u}$ is unit vector in the direction of $\vec{B}$

$$= \|\vec{A}\| \left\| \frac{\vec{B}}{\|\vec{B}\|} \right\| \cos\theta \; \frac{\vec{B}}{\|\vec{B}\|}$$

$\Updownarrow$ definition of dot product

$$= (\vec{A} \bullet \hat{u}) \; \hat{u}$$

a) $\quad (A \cdot \hat{u})\, \hat{u} = -2\vec{j}$

$\quad \| \quad \mathrm{proj}_{\vec{j}} \vec{A}$

b) $\quad \hat{u} = \dfrac{\vec{B}}{\|\vec{B}\|} = \dfrac{-\vec{i} + 4\vec{k} + 3\vec{j}}{\sqrt{26}}$

$$\mathrm{proj}_{\vec{B}} \vec{A} = \frac{-1-6-4}{\sqrt{26}} \; \hat{u}$$

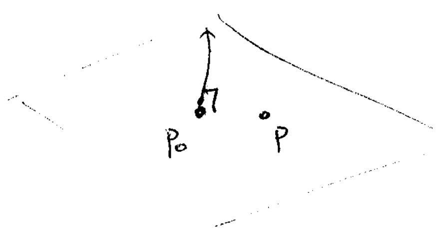
$$= \frac{-11}{\sqrt{26}} \; \hat{u}$$

(5) Using previous formula (problem 4)

we get $\quad \vec{A} = \vec{B} \quad$ or

$\vec{A} \perp \vec{B} \quad$ i.e. $\quad \vec{A} \bullet \vec{B} = 0$

(6)



$P_0 = (1, 1, 3)$

$P = (x, y, z)$

$\vec{N} = 3\vec{i} + 4\vec{j} + 5\vec{k}$

$$\overrightarrow{P_0 P} \bullet \vec{N} = 0$$

$$3(x-1) + 4(y-1) + 5(z-3) = 0$$

vector parallel to plane dot with a perpendicular vector must be 0

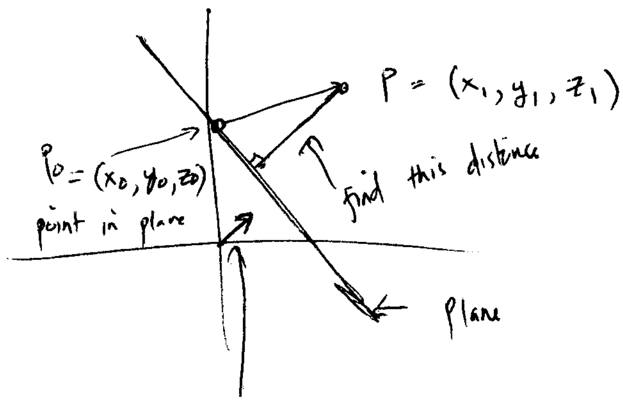
a) $\quad 3x + 4y + 5z - 22 = 0$

b)

⑥ b) using the formula $\dfrac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2+B^2+C^2}}$ where (x_1, y_1, z_1) is the point we want to find the distance from, we get

$$\text{Distance} = \frac{|6+32+20-22|}{\sqrt{3^2+4^2+5^2}} = \frac{36}{\sqrt{50}} = \frac{36}{5\sqrt{2}} \text{ units}$$

if you do not remember this formula, you can still derive it



$\vec{N}$ perpendicular vector to plane (read off from plane eqn)

$$\vec{N} = A\vec{i} + B\vec{j} + C\vec{k}$$

take projection of $\overrightarrow{P_0P}$ onto $\vec{N}$, i.e. $\text{proj}_{\vec{N}} \overrightarrow{P_0P}$

and look at its length

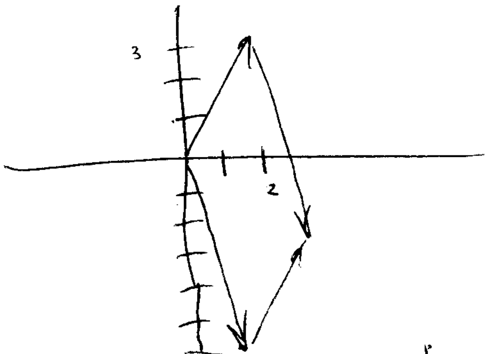
$$\left\| \text{proj}_{\vec{N}} \overrightarrow{P_0P} \right\| = \Bigg\| \underbrace{\left(\overrightarrow{P_0P} \cdot \frac{\vec{N}}{\|\vec{N}\|} \right)}_{\text{scalar}} \frac{\vec{N}}{\|\vec{N}\|} \Bigg\| = \frac{|\overrightarrow{P_0P} \cdot \vec{N}|}{\|\vec{N}\|}$$

$$= \frac{|A(x_1-x_0) + B(y_1-y_0) + C(z_1-z_0)|}{\sqrt{A^2+B^2+C^2}} = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2+B^2+C^2}}$$

7

b) $\left\| \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & 0 \\ 1 & -6 & 0 \end{vmatrix} \right\| = \left\| \vec{k}(-12-3) \right\| = 15$ square units

a)



8

a) $A \times B = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 3 & 4 \\ 2 & 1 & 1 \end{vmatrix} = -\vec{i} + 9\vec{j} - 5\vec{k}$

check this perpendicular to $\vec{A}, \vec{B}$

b) $\| A \times B \| =$ Area of Parallelogram

$\| = \sqrt{107}$ square units

c) $| A \cdot (B \times C) | =$ Volume of parallelipiped

d) Look it up

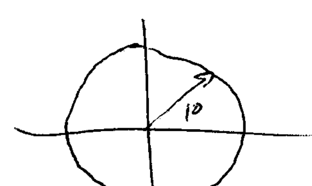
9

a) $\vec{G}'(\frac{\pi}{4}) = -10 \sin\frac{\pi}{4}\,\vec{i} + 10 \cos\frac{\pi}{4}\,\vec{j}$

$= -5\sqrt{2}\,\vec{i} + 5\sqrt{2}\,\vec{j}$

b) $\| \vec{G}(t) \| = \sqrt{(10\cos t)^2 + (10 \sin t)^2} = 10$

c) a circle of radius 10



d) $\vec{G}(t) \cdot \vec{G}'(t) = (10\cos t)(-10 \sin t) + (10 \sin t)(10 \cos t) = 0$

e) $\pi/2$ since $\vec{G} \cdot \vec{G}' = 0$ i.e. perpendicular