Solution Quiz 8 4pm

1. $w = xy + \ln(z)$, $x = \frac{v^2}{u}$, $y = u+v$, $z = \cos(u)$

Find $\frac{\partial w}{\partial u}$

$$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial u} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial u} = y\frac{\partial}{\partial u}\left(\frac{v^2}{u}\right) + x\frac{\partial}{\partial u}(u+v) + \frac{1}{z}\frac{\partial}{\partial u}(\cos(u)) =$$

$$= yv^2\frac{\partial}{\partial u}\left(\frac{1}{u}\right) + x + \frac{1}{z}(-\sin u) = yv^2\left(\frac{-1}{u^2}\right) + x - \frac{\sin(u)}{z}$$

substitute for the intermidiate variables

$$= \frac{-(u+v)v^2}{u^2} + \frac{v^2}{u} - \frac{\sin(u)}{\cos(u)}.$$

2. Find $\left(\frac{df}{ds}\right)_{u,P_0}$ for $f(x,y) = 2x^2 + 2y^2$

$P_0(-1,1)$ $A = 3i - 4j$

$$u = \frac{A}{|A|} = \frac{3i-4j}{\sqrt{9+16}} = \frac{3i-4j}{5} = \frac{3}{5}i - \frac{4}{5}j$$ then $u_1 = 3/5$, $u_2 = -4/5$, $x_0 = -1$, $y_0 = 1$

$$\left(\frac{df}{ds}\right)_{u,P_0} = \lim_{s\to 0}\frac{f(x_0+su_1, y_0+su_2) - f(x_0,y_0)}{s} =$$

$$= \lim_{s\to 0}\frac{f(-1+s.3/5,\ 1-s.4/5) - f(-1,1)}{s} = \lim_{s\to 0}\frac{2(-1+\frac{3}{5}s) + 2(1-\frac{4}{5}s) - (2+2)}{s}$$

$$= \lim_{s\to 0}\frac{-2 + 6/5\,s + 2 - 8/5\,s - 4}{s} = \lim_{s\to 0}\frac{-\frac{2}{5}s - 4}{s} = \lim_{s\to 0}\left(-\frac{2}{5}\right) - \lim_{s\to 0}\frac{4}{s}$$

$$= -\frac{2}{5}$$