Instr.: Woei *May 25, 2006*

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Show all work and justifications to receive full credit.
No Calculators.

1. *(6 pts)*
By considering different paths of approach, show that the function $f(x,y) = \frac{x^2}{x^2-y}$ has no limit as $(x,y) \to (0,0)$. Hint: Consider lines or parabolas that pass through the origin.

The curve $y = 2x^2$ & $y = -2x^2$ pass through the origin.

so the $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ along $y = 2x^2$ is

$$\lim_{x\to 0} \frac{x^2}{x^2 - 2x^2} = \lim_{x\to 0} \frac{x^2}{-x^2} = \lim_{x\to 0} -1 = -1 \qquad (1)$$

and the $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ along $y = -2x^2$ is

$$\lim_{x\to 0} \frac{x^2}{x^2+2x^2} = \lim_{x\to 0} \frac{x^2}{3x^2} = \lim_{x\to 0} \frac{1}{3} = \frac{1}{3} \qquad (2).$$

Since (1) & (2) are not equal thus by the Two Path Limit Test, the limit does not exist.

2. *(4 pts)* Find f_x, f_y, f_{xy}, and f_{xx} for the function $f(x,y) = 2\sin(xy)$.

$$f_x = \frac{\partial}{\partial x} f(x,y) = 2\cos(xy)\, y$$

y is held constant

$$f_y = 2\cos(xy)\, x$$

$$f_{xy} = (f_x)_y = (2\cos(xy)\, y)_y = 2\left[\cos(xy) - \sin(xy)\, xy\right]$$

using product rule & holding x constant

$$f_{xx} = -2\sin(xy)\, y^2$$