6-7pm | Quiz 7

① By considering different paths of approach, show that the function f has no limit as $(x,y)\to(0,0)$, where

$$f(x,y)=\frac{x+y}{x-y}$$

Sol.

If $y=0$, then

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{x\to0}\frac{x}{x} = \lim_{x\to0} 1 = 1$$

On the other hand, if $y=-x$, then

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{x\to0}\frac{x+(-x)}{x-(-x)} = \lim_{x\to0}\frac{0}{2x} = \lim_{x\to0} 0 = 0$$

② Find f_x, f_y, f_{xy}, f_{xx}, for the function $f(x,y)=e^{xy}$

Sol.

$$f_x(x,y)=\frac{\partial}{\partial x}f(x,y)=e^{xy}\cdot\frac{\partial}{\partial x}(xy)=e^{xy}\cdot y$$

$$f_x(x,y)=\frac{\partial}{\partial y}f(x,y)=e^{xy}\cdot x \quad \text{(Similarly)}$$

$$f_{xy}(x,y)=\frac{\partial}{\partial y}(f_x)(x,y)=\frac{\partial}{\partial y}(e^{xy}\cdot y)=e^{xy}+x\cdot e^{xy}\cdot y=e^{xy}(1+xy)$$

$$f_{xx}(x,y)=\frac{\partial}{\partial x}(f_x)(x,y)=\frac{\partial}{\partial x}(e^{xy}\cdot y)=y^2e^{xy}$$