5-6pm | QUIZ 7

① By considering different paths of approach show that the function f has no limit as $(x,y)\to(0,0)$, where $f(x,y) = \dfrac{xy}{|xy|}$

Solution

Put $x=1$ then

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{y\to 0} \frac{y}{|y|} = \begin{cases} +1 \\ -1 \end{cases}$$

since $\dfrac{y}{|y|} = \begin{cases} 1 & \text{if } y>0 \\ -1 & \text{if } y<0 \end{cases}$

② Find f_x, f_y, f_{xy}, f_{xx}, for the function $f(x,y) = \cos(xy)$

Sol.

$$\frac{\partial f}{\partial x}: f_x(x,y) = -\sin(xy)\cdot\frac{\partial}{\partial x}(xy) = -\sin(xy)\cdot y$$

$$\frac{\partial f}{\partial y}: f_y(x,y) = -\sin(xy)\cdot x \quad \text{(similarly)}$$

$$f_{xy}(x,y) = \frac{\partial}{\partial y}(f_x)(x,y) = \frac{\partial}{\partial y}\left(-\sin(xy)\cdot y\right) = -\frac{\partial}{\partial y}(\sin(xy)\cdot y)$$

(prod. rule)

$$= -\left[\sin(xy) + y\cos(xy)\,x\right]$$

$$= -\sin(xy) - x\cdot y\cdot\cos(xy)$$

$$f_{xx}(x,y) = \frac{\partial}{\partial x} f_x(x,y) = -y^2\cos(xy)$$