Quiz 7 Solutions

1: consider $y = kx^2$ (parabola's), $f(x,y) = \frac{x^4}{x^4+y^2}$

$$f(x,y)\Big|_{y=kx^2} = \frac{x^4}{x^4+y^2}\Big|_{y=kx^2} = \frac{x^4}{x^4+(kx^2)^2} = \frac{x^4}{x^4+k^2x^4} = \frac{1}{k^2}$$

then

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} \left(f(x,y)\Big|_{y=kx^2} \right) = \lim_{(x,y)\to(0,0)} \frac{1}{k^2} = \frac{1}{k^2}$$

The limit varies with the path of approach.
If (x,y) approaches $(0,0)$ along the parabola $y=x^2$, $k=1$ so limit $=1$
If (x,y) approaches $(0,0)$ along $y=2x^2$, $k=2$ so limit $=\frac{1}{4}$

2: $f(x,y) = \sin(x\cdot y)$

$$f_x = \frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(\sin(xy)) = \cos(xy)\cdot y$$

$$f_y = \frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(\sin(xy)) = \cos(xy)\cdot x$$

$$f_{xy} = \frac{\partial}{\partial y}(f_x) = \frac{\partial}{\partial y}(\cos(xy)\cdot y) = \left[\frac{\partial}{\partial y}\cos(xy)\right]\cdot y + \cos(xy)\frac{\partial}{\partial y} y$$

$$= -\sin(xy)\, xy + \cos(xy)$$

$$f_{yx} = \frac{\partial}{\partial x}(f_y) = \frac{\partial}{\partial x}(\cos(xy)\cdot x) = \left[\frac{\partial}{\partial x}\cos(xy)\right]\cdot x + \cos(xy)\cdot\frac{\partial}{\partial x} x$$

$$= -\sin(xy)\cdot yx + \cos(xy).$$