Name: Key *Student ID:*_____

No Calculators.
Show all work and justifications to receive full credit.

1. (*4 pts*) Find the angle between the two planes $7x + 7y + 7z = 7$ and $3x - 10y + 5z = -8$. You do not need to simplify your answer.

To find the angle between two planes is equivalent to finding the angle between the normals to the plane.

$\langle 7, 7, 7 \rangle$ or $\langle 1, 1, 1 \rangle = n_1$ is the normal to plane $7x + 7y + 7z = 7$ & $\langle 3, -10, 5 \rangle = n_2$ is the normal to plane $3x - 10y + 5z = -8$. Thus $\cos\theta = \dfrac{n_1 \cdot n_2}{\|n_1\| \, \|n_2\|}$

$= \dfrac{-2}{\sqrt{3}\sqrt{134}} = \dfrac{-2}{\sqrt{402}} \Rightarrow \theta = \cos^{-1}\left(\dfrac{-2}{\sqrt{402}}\right)$

2. (*4 pts*) Find the domain and range of $f(x,y) = \frac{1}{\sqrt{7-x^2-y^2}}$.

The domain $(f) = \{(x,y) \mid 7 - x^2 - y^2 > 0\} = \{(x,y) \mid x^2 + y^2 < 7\}$

= points in the circle of radius $\sqrt{7}$ centered at the origin

range $(f) = \left[\frac{1}{\sqrt{7}}, \infty\right)$ since the largest $7 - x^2 - y^2$ is 7 & smallest is close to 0.

3. (*2 pts*) Describe the level curve $1 = z = f(x,y)$ where $f(x,y)$ is defined in problem 2.

$1 = \dfrac{1}{\sqrt{7 - x^2 - y^2}} \Rightarrow \sqrt{7 - x^2 - y^2} = 1 \Rightarrow 7 - x^2 - y^2 = 1$

$\Rightarrow x^2 + y^2 = 6$ which is a circle of radius $\sqrt{6}$ centered at the origin.