Solutions Quiz 6 5 pm

1. $P_1: 5x+5y+5z=5$ $P_2: x-8y+3z=-6$

the angle between P_1 and P_2 is the angle between their normal vectors n_1 and n_2.

$n_1 = \langle 5, 5, 5 \rangle$ $n_2 = \langle 1, -8, 3 \rangle$

$$\theta = \cos^{-1}\left(\frac{n_1 \cdot n_2}{|n_1| \cdot |n_2|}\right) = \cos^{-1}\left(\frac{\langle 5,5,5\rangle \cdot \langle 1,-8,3\rangle}{\sqrt{25+25+25}\,\sqrt{1+64+9}}\right) =$$

$$= \cos^{-1}\left(\frac{5-40+15}{\sqrt{3(25)}\,\sqrt{74}}\right) = \cos^{-1}\left(\frac{-20}{5\sqrt{3}\,\sqrt{74}}\right) = \cos^{-1}\left(\frac{-20}{5\sqrt{222}}\right)$$

$$= \cos^{-1}\left(\frac{-4}{\sqrt{222}}\right)$$

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2. $f(x,y) = \dfrac{1}{\sqrt{5-x^2-y^2}}$

since division by zero and square root of a negative number are undefined, then the domain consists of all points (x,y) where $5-x^2-y^2$ is non zero and non-negative, in other words all points (x,y) where $5-x^2-y^2 > 0$ which is the same as $5 > x^2+y^2$.

Therefore Domain$(f) = \{(x,y) : x^2+y^2 < 5\}$

[the points in the plane which are inside the circle centered at the origin with radius $\sqrt{5}$.]

Solving $a = \dfrac{1}{\sqrt{5-x^2-y^2}}$ we get $5 - \dfrac{1}{a^2} = x^2+y^2$ which makes sense iff $5 - \dfrac{1}{a^2} \geq 0$ iff $a \geq \dfrac{1}{\sqrt{5}}$, Range f: $z \geq \dfrac{1}{\sqrt{5}}$

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3. level curve $1 = \dfrac{1}{\sqrt{5-x^2-y^2}}$

equivalent to: $\sqrt{5-x^2-y^2} = 1$

$5-x^2-y^2 = 1$

$4 = x^2+y^2$

then the level curve $f(x,y)=1$ is a circle centered at the origin and radius 2