Solutions Quiz 6 4pm

1: $P_1: 4x+4y+4z=4$ $P_2: -7y+2z=-5$

angle between P_1 and P_2 is the angle between their normal vectors n_1 and n_2.

$n_1 = \langle 4,4,4\rangle$ $n_2 = \langle 0,-7,2\rangle$

$$\theta = \cos^{-1}\left(\frac{n_1\cdot n_2}{|n_1||n_2|}\right) = \cos^{-1}\left(\frac{\langle 4,4,4\rangle\cdot\langle 0,-7,2\rangle}{\sqrt{16+16+16}\sqrt{0+49+4}}\right) =$$

$$= \cos^{-1}\left(\frac{-28+8}{\sqrt{3(16)}\sqrt{53}}\right) = \cos^{-1}\left(\frac{-20}{4\sqrt{3}\sqrt{53}}\right) = \cos^{-1}\left(\frac{-20}{4\sqrt{159}}\right)$$

$$= \cos^{-1}\left(\frac{-5}{\sqrt{159}}\right)$$

2: $$f(x,y) = \frac{1}{\sqrt{4-x^2-y^2}}$$

since division by zero and square root of a negative are undefined, then the domain of f consists of all points (x,y) where $4-x^2-y^2$ is non-zero and non-negative, in other words all points (x,y) where $4-x^2-y^2>0$, which is the same as $4>x^2+y^2$.

$$\text{Domain}(f) = \{(x,y) : x^2+y^2<4\}$$

[points in the plane which are inside the circle centered at the origin with radius 2]

Solving $a = \dfrac{1}{\sqrt{4-x^2-y^2}}$ we get $4-\dfrac{1}{a^2} = x^2+y^2$ which makes sense $a \geq 1/2$

Therefore the range of f is : $z \geq 1/2$

3: level curve $1 = \dfrac{1}{\sqrt{4-x^2-y^2}}$

equivalent to: $\sqrt{4-x^2-y^2} = 1$

$4-x^2-y^2 = 1$

$3 = x^2+y^2$

then the level curve $f(x,y)=1$ is a circle centered at the origin and radius $\sqrt{3}$