Quiz 5 | 6:00pm – 7:00pm

① How close is the approximation $\sin x = x$ when $|x| < 10^{3}$?

② Describe the sets of points in the space whose coordinates satisfy the given inequalities (or combination of equations and inequalities)

$$x^2+y^2+z^2 \leq 1$$

Solutions

① The series of $\sin x$ can be written

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

This series is an alternating series for every nonzero value of x. According to the alternating series estimation theorem the error in truncating

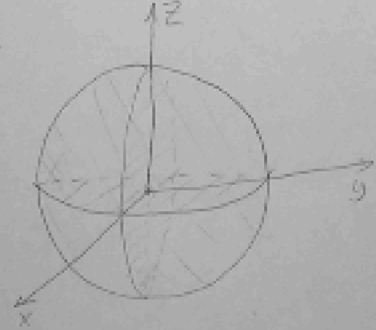
$$\sin x = x \;\vdots\; - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

after x is not greater than $\left|-\frac{x^3}{3!}\right| = \frac{|x|^3}{3!} = \frac{|x|^3}{6}$

Thus, since $|x| < 10^{3}$, we have $\frac{|x|^3}{6} < \frac{(10^3)^3}{6} = \frac{10^1}{6} = \frac{1}{60}$

meaning that the approximation when $|x| < 10^{3}$ has error less than $1/60$.

② $A = \{(x,y,z) \mid x^2+y^2+z^2 \leq 1\}$ is the solid ball with center $(0,0,0)$ and radius 1.



$A_1 = \{(x,y,z) \mid x^2+y^2+z^2 = 1\}$ is the sphere of radius 1 and center $(0,0,0)$.

$A_2 = \{(x,y,z) \mid x^2+y^2+z^2 < 1\}$ is the ball with radius strictly less than 1 and center $(0,0,0)$

$A = A_1 \cup A_2$