Quiz #5 | 5:00 pm - 6:00 pm

① Find the Taylor series at $a=0$ for $\cos^2 x$

Hint: Consider the identity $\cos^2 x = \frac{(1+\cos 2x)}{2}$

② Give a geometric description of the set of points in the space whose coordinates satisfy the given pairs of equations

$$x^2+y^2+z^2=25 \quad \cdots (1)$$
$$y=-4 \quad \cdots (2)$$

Solutions

① Recall the Taylor series for cosine about the point $a=0$ is given by

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

so

$$\cos 2x = \sum_{n=0}^{\infty} \frac{(-1)^n (2x)^{2n}}{(2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n}}{(2n)!} x^{2n}$$

This implies that

$$\cos^2 x = \frac{1}{2}\left(1 + \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n} x^{2n}}{(2n)!}\right) = \frac{1}{2} + \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n-1}}{(2n)!} x^{2n}$$

$$= \frac{1}{2} + \frac{1}{2} + \sum_{n=1}^{\infty} \frac{(-1)^n 2^{2n-1}}{(2n)!} x^{2n} = 1 + \sum_{n=1}^{\infty} \frac{(-1)^n 2^{2n-1}}{(2n)!} x^{2n}$$

$$= 1 - \frac{2}{2!}x^2 + \frac{2^3}{4!}x^4 - \frac{2^5}{6!}x^6 + \ldots$$

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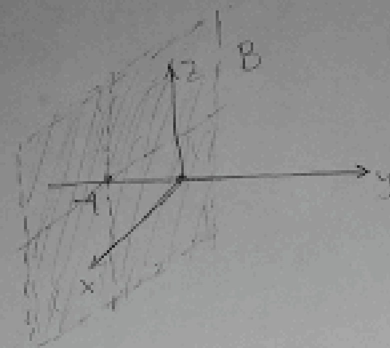
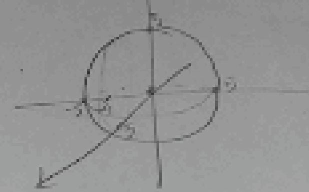
② $A = \{(x,y,z) \mid x^2+y^2+z^2 = 25 = 5^2\}$ corresponds to the points in a sphere of radius 5 and center $(0,0,0)$.

$B = \{(x,y,z) \mid y=-4\}$ corresponds to a plane passing through the point $(0,-4,0)$ which is parallel to the x-z plane.





So the set of points satisfying both equations (1) and (2), are the points (x,y,z) such that

$$x^2+(-4)^2+z^2=5^2$$

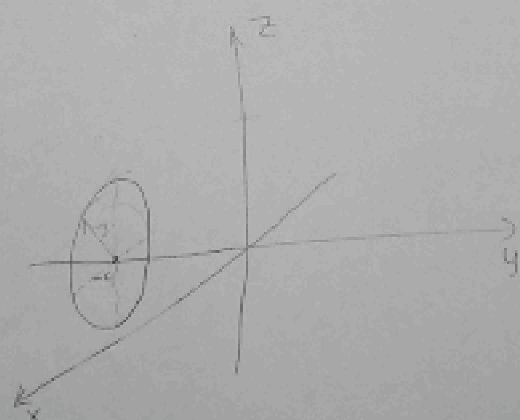
$$\iff \quad x^2+z^2=25-16=9=3^2$$

$$\iff \quad x^2+z^2=3^2$$

That is, the points in the set

$$C=\{(x,-4,z) \mid x^2+z^2=3^2\}$$

i.e., the circle in $\mathbb{R}^3$ with center $(0,-4,0)$ and radius 3.



Notice that $C=A\cap B$.