| Quiz 4 | 5:00 pm – 6:00 pm

The series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \qquad \cdots \quad (1)$$

converges to e^x for all x

a) Find the series for $\frac{d}{dx}(e^x)$. What series did you get? Explain. What's its interval of convergence.

b) Find the series for $\int e^x dx$. What series did you get? Explain.

Solution

By theorem 19 of textbook

a) $$\frac{de^x}{dx} = \frac{d}{dx}(1) + \frac{d}{dx}\left(\frac{x}{1!}\right) + \cdots + \frac{d}{dx}\left(\frac{x^n}{n!}\right) + \frac{d}{dx}\left(\frac{x^{n+1}}{(n+1)!}\right) + \cdots$$

$$= 0 + 1 + \frac{\cancel{2}x}{\cancel{2}(1)} + \frac{\cancel{3}x^2}{\cancel{3}\cdot 2\cdot 1} + \frac{\cancel{4}x^3}{\cancel{4}\cdot 3\cdot 2\cdot 1} + \cdots + \frac{\cancel{n}x^{n-1}}{\cancel{n}(n-1)\cdots(2)(1)} + \frac{\cancel{(n+1)}x^n}{\cancel{(n+1)}(n)(n-1)\cdots(2)(1)} + \cdots$$

$$= 1 + \frac{x}{1} + \frac{x^2}{2\cdot 1} + \frac{x^3}{3\cdot 2\cdot 1} + \cdots + \frac{x^{n-1}}{(n-1)!} + \frac{x^n}{n!} + \cdots$$

Observe that this series corresponds exactly to the series of e^x in (1). (We really were expecting that since we know that $\frac{d}{dx}(e^x) = e^x$). The interval of convergence is all the real numbers as the problem states.

By theorem 20 of textbook

b) $$\int e^x dx = \int 1 dx + \int \frac{x^2}{2!} dx + \int \frac{x^3}{3!} dx + \cdots + \int \frac{x^{n-1}}{(n-1)!} dx + \int \frac{x^n}{n!} dx + \cdots$$

$$= \left(x + \frac{x^3}{3(2!)} + \frac{x^4}{4(3!)} + \cdots + \frac{x^n}{n(n-1)!} + \frac{x^{n+1}}{(n+1)n!} + \cdots\right) + C$$

Observe that this is the general antiderivative of e^x, i.e., this is the function e^x up to a constant term.