Quiz 3

The series $\sec x = 1 + \frac{x^2}{2} + \frac{5}{24}x^4 + \frac{61}{720}x^6 + \frac{277}{8064}x^8 + \dots$

converges to $\sec x$ for $-\pi/2 < x < \pi/2$

(a) Find the first five terms of a power series for the function $\ln|\sec x + \tan x|$. For what values of x should the series converge?

$\boxed{\text{Solution}}$ we are going to use "the term-by-term Integration theorem"

$$\int \sec x \, dx = \ln|\sec x + \tan x| + C$$

and also:

$$\int \sec x \, dx = \int \left(1 + \frac{x^2}{2} + \frac{5}{24}x^4 + \frac{61}{720}x^6 + \frac{277}{8064}x^8 + \dots\right)dx =$$

$$= x + \frac{x^3}{6} + \frac{5x^5}{5 \cdot 24} + \frac{61}{7(720)}x^7 + \frac{277}{9(8064)}x^9 + \dots + C$$

when $x = 0 \Rightarrow C = 0$

Therefore $\ln|\sec x + \tan x| = x + \frac{x^3}{6} + \frac{x^5}{24} + \frac{61}{5040}x^7 + \frac{277}{72576}x^9 + \dots$

converges for $-\pi/2 < x < \pi/2$

(b) Find the first four terms of a series for $\sec x \tan x$

$\boxed{\text{Solution}}$ we are going to use "the term-by-term differentiation theorem"

$$\frac{d}{dx}\sec x = \sec x \tan x$$

also:

$$\frac{d}{dx}\sec x = \frac{d}{dx}\left(1 + \frac{x^2}{2} + \frac{5}{24}x^4 + \frac{61}{720}x^6 + \frac{277}{8064}x^8\right) =$$

$$= 0 + \frac{2x}{2} + \frac{5 \cdot 4}{24}x^3 + \frac{6 \cdot 61}{720}x^5 + \frac{8 \cdot 277}{8064}x^7 = x + \frac{5}{6}x^3 + \frac{61}{120}x^5 + \frac{277}{1008}x^7 + \dots$$