Instr.: Woei *April 20, 2006*

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No Calculators.

1. (*5 pts*) Does the series

$$\sum_{n=1}^{\infty} \frac{1}{1+\ln^2 n} \qquad a_n = \frac{1}{1+\ln^2 n}$$

converge or diverge? Give reasons for your answers.

Recall Limit Comparison Test (L.C.T.) Part 3. which says

$\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$ (a_n is dominant) & $\sum_{n=1}^{\infty} b_n$ diverges then $\sum_{n=1}^{\infty} a_n$ diverges

If we choose $b_n = \frac{1}{n}$ then $\lim_{n\to\infty} \frac{\frac{1}{1+\ln^2 n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n}{1+\ln^2 n} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{\underset{\text{L'hopital}}{=}}$

$= \lim_{n\to\infty} \frac{1}{2\ln n \frac{1}{n}} = \lim_{n\to\infty} \frac{n}{2\ln n} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{\underset{\text{L'hopital}}{=}} \lim_{n\to\infty} \frac{1}{2\frac{1}{n}} = \lim_{n\to\infty} \frac{n}{2} = \infty$

Therefore $\sum_{n=1}^{\infty} \frac{1}{1+\ln^2 n}$ diverges

2. (*5 pts*) Does the series

$$\sum_{n=1}^{\infty} (-1)^n \frac{(n!)^2\, 3^n}{(2n+1)!}$$

converge absolutely, converge, or diverge? Give reasons for your answers.

Let $a_n = \frac{(-1)^n (n!)^2 3^n}{(2n+1)!}$. To see if this series converges absolutely we check if $\sum |a_n|$ converges. We will use ratio to do this.

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left| \frac{(-1)^{n+1} ((n+1)!)^2 3^{n+1}}{(2(n+1)+1)!} \frac{(2n+1)!}{(-1)^n (n!)^2 3^n} \right| = \lim_{n\to\infty} \frac{(n+1)^2}{(2n+3)(2n+2)}$

$= \lim_{n\to\infty} \frac{n^2+2n+1}{4n^2+10n+6} \frac{\left(\frac{1}{n^2}\right)}{\left(\frac{1}{n^2}\right)} = \lim_{n\to\infty} \frac{1+\frac{2}{n}+\frac{1}{n^2}}{4+\frac{10}{n}+\frac{6}{n^2}} = \frac{1}{4} < 1$

therefore $\sum_{n=1}^{\infty} a_n$ converges absolutely & therefore $\sum_{n=1}^{\infty} a_n$ converges.