Name: Key *Student ID:*

No Calculators.

1. (*5 pts*) Does the series

$$\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n^{3/2}}$$

converge or diverge? Give reasons for your answers.

Recall : Limit Comparison Test (L.C.T.) part 2. which says

If $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$ & $\sum_{n=1}^{\infty} b_n$ converges Then $\sum_{n=1}^{\infty} a_n$ converges.

Here $a_n = \frac{(\ln n)^2}{n^{3/2}}$ and we will choose $b_n = \frac{1}{n^{5/4}}$ $(p = 5/4 > 1)$

So, $\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{(\ln n)^2}{n^{1/4}} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{\underset{\text{L'hopital's}}{=}} \lim_{n\to\infty} \frac{2 \ln n \, \frac{1}{n}}{\frac{1}{4} n^{-3/4}} = 8 \lim_{n\to\infty} \frac{\ln n}{n^{1/4}} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{\underset{\text{L'hopital}}{=}}$

$= 8 \lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{1}{4n^{3/4}}} = 32 \lim_{n\to\infty} \frac{n^{3/4}}{n} = 0$. Thus since $\sum_{n=1}^{\infty} \frac{1}{n^{5/4}}$ converges

Therefore $\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n^{3/2}}$ converges.

2. (*5 pts*) Does the series

$$\sum_{n=1}^{\infty} \frac{(n!)^n}{n^{n^2}}$$

converge or diverge? Give reasons for your answers.

The natural test to use is Root Test since n is the exponent.

$a_n = \frac{(n!)^n}{n^{n^2}}$ so $\lim_{n\to\infty} \sqrt[n]{\frac{(n!)^n}{n^{n^2}}} = \lim_{n\to\infty} \frac{n!}{n^n}$

Recall $\frac{n!}{n^n} = \frac{1 \cdot 2 \cdot 3 \cdots n}{n \cdot n \cdots n} < \frac{1}{n}$ and $\lim_{n\to\infty} \frac{1}{n} = 0$

therefore $\lim_{n\to\infty} \frac{n!}{n^n} = 0$ so by Root Test

$\sum_{n=1}^{\infty} \frac{(n!)^n}{n^{n^2}}$ converges.