11.4.25
11.6.20

Quiz 3

5:00-6:00

1) Determine if the following series diverges or converges

$$\sum_{n=1}^{\infty} \sin\frac{1}{n}$$

$a_n = \sin\frac{1}{n}$
$b_n = \frac{1}{n}$

Solution

Since $\lim\limits_{n\to\infty} \dfrac{\sin\frac{1}{n}}{\frac{1}{n}} = 1$ and $\sum\limits_{n=1}^{\infty} \frac{1}{n}$ diverges

then by limit comparison test(1) $\sum\limits_{n=1}^{\infty} \sin\frac{1}{n}$ diverges

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2) Determine convergence, divergence or absolute convergence

$$\sum_{n=2}^{\infty} (-1)^n \frac{1}{\ln(n^3)}$$

Observe that:
$\ln(n) \leq \ln(n+1)$

Solution

This series converges:

Using the alternating series test, it is clear that $1/\ln(n^3) > 0$

$1/\ln(n^3) = 1/3\ln n$ is decreasing $\left(1/3\ln(n) \geq 1/3\ln(n+1)\right)$

and $\lim\limits_{n\to\infty} \dfrac{1}{3\ln n} = 0$. Thus the series converges. But

the series $\sum\limits_{n=2}^{\infty} \left| (-1)^n \dfrac{1}{\ln(n^3)} \right| = \sum\limits_{n=2}^{\infty} \dfrac{1}{\ln(n^3)}$ does not converge because $3\ln(n) \leq 3n$, $\Rightarrow \frac{1}{3\ln(n)} \geq \frac{1}{3n} \Rightarrow \sum_{n=2}^{\infty}(3\ln(n))^{-1} \geq \sum_{n=2}^{\infty}(3n)^{-1}$

~~and~~ $\sum_{n=2}^{\infty} \frac{1}{3n}$ does not converge

$\left(\Rightarrow \sum_{n=2}^{\infty} \frac{1}{3\ln(n)} = \sum_{n=2}^{\infty} \frac{1}{\ln(n^3)} \text{ does not converge.} \right)$

Thus $\sum_{n=2}^{\infty}(-1)^n \frac{1}{\ln(n^3)}$ does not converge absolutely.

Therefore the series converges conditionally.