Name: Key *Student ID:*_____

1. (*5 pts*) Does the series

$$\sum_{n=1}^{\infty} \ln \frac{1}{n}$$

converge or diverge? Give reason for your answers. If this series converges, find its sum.

Looking at the terms in the sum, $a_n = \ln \frac{1}{n}$, we see that $a_n \to -\infty$ as $n \to \infty$ since $\frac{1}{n} \to 0$ as $n \to \infty$ therefore by the nth term test $\sum_{n=1}^{\infty} \ln \frac{1}{n}$ diverges //

2. (*5 pts*) Express the number below as the ratio of two integers.

$$1.24\overline{123} = 1.24\ 123\ 123\ 123\ \ldots$$

$$1.24\overline{123} = 1.24 + 0.0\overline{123} = \frac{124}{100} + \frac{123}{10^5} + \frac{123}{10^8} + \frac{123}{10^{11}} + \frac{123}{10^{14}} + \cdots$$

$$= \frac{124}{100} + \frac{1}{10^5}\left(123 + \frac{123}{10^3} + \frac{123}{10^6} + \frac{123}{10^9} + \cdots\right)$$

$$= \frac{124}{100} + \frac{1}{10^5}\left(\sum_{n=0}^{\infty} \frac{123}{10^{3n}}\right) \leftarrow \text{geometric series}$$

$124(1000-1) = 123876$

$$= \frac{124}{100} + \frac{1}{10^5} \cdot \frac{123}{1 - \frac{1}{10^3}}$$

$$= \frac{124}{100} + \frac{123}{99900} = \frac{124 \cdot 999 + 123}{99900}$$

$$= \frac{123999}{99900} //$$