| QUIZ 2 | 6:00–7:00pm

① Determine if the sequence is nondecreasing and if it is bounded from above

$a_n = \frac{3n+1}{n+1}$

First of all notice that $\lim_{n\to\infty} a_n = 3$. Let us calculate the first five terms of this sequence

$$a_1 = 2 = \frac{60}{30}$$
$$a_2 = \frac{7}{3} = \frac{70}{30}$$
$$a_3 = \frac{10}{4} = \frac{5}{2} = \frac{75}{30}$$
$$a_4 = \frac{13}{5} = \frac{78}{30}$$
$$a_5 = \frac{16}{6} = \frac{80}{30}$$
$$\vdots$$

So $a_1 < a_2 < a_3 < a_4 < a_5$

Claim: $a_n \leq a_{n+1}$ for all n

Suppose that $a_{n+1} \geq a_n \Rightarrow \frac{3(n+1)+1}{(n+1)+1} \geq \frac{3n+1}{n+1} \Rightarrow \frac{3n+4}{n+2} \geq \frac{3n+1}{n+1}$

$$\Rightarrow 3n^2+3n+4n+4 \geq 3n^2+6n+n+2$$
$$\Rightarrow 3n^2+7n+4 \geq 3n^2+7n+2$$

But all the steps here are reversible (every $\Rightarrow$ symbol can be replaced by $\Leftarrow$, moreover can be replaced by $\Leftrightarrow$). Thus $\{a_n\}$ is nondecreasing.

To show that $\{a_n\}$ is bounded from above let us use the theorem 6 (page 740):

A nondecreasing sequence of real numbers converges if and only if it is bounded from above. From above we know that $\{a_n\}$ is nondecreasing, so it remains to check that $\{a_n\}$ converges, that will imply that $\{a_n\}$ is bounded from above

$$\lim_{n\to\infty} \frac{3n+1}{n+1} = \lim_{n\to\infty} \frac{3+\frac{1}{n}}{1+\frac{1}{n}} = \frac{3}{1} = 3 \text{ (exists!)}$$

(2) Determine convergence or divergence of the series

$$\sum_{n=1}^{\infty} \frac{\operatorname{Tan}^{-1} n}{n^{1.1}}$$

First of all observe that

$$\operatorname{Tan}^{-1} n \leq \pi/2 \text{ , for all } n$$

Thus $$\sum_{n=1}^{\infty} \frac{\operatorname{Tan}^{-1} n}{n^{1.1}} \leq \sum_{n=1}^{\infty} \frac{\pi/2}{n^{1.1}} = \frac{\pi}{2} \underbrace{\sum_{n=1}^{\infty} \frac{1}{n^{1.1}}}_{\text{p-series with } p=1.1}$$

A p-series converges if $p>1$. Thus $\sum_{n=1}^{\infty} \frac{\operatorname{Tan}^{-1} n}{n^{1.1}}$ converges