QUIZ 2 | 5:00pm-6:00pm

① Find the sum of the series $\sum_{n=0}^{\infty} \frac{5\cdot 3^n - 2^n}{3^n \cdot 2^n}$

$$\sum_{n=0}^{\infty}\left(\frac{5\cdot 3^n-2^n}{3^n\cdot 2^n}\right)=\sum_{n=0}^{\infty}\left(5\left(\frac{1}{2}\right)^n-\left(\frac{1}{3}\right)^n\right)$$

$$=5\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n-\sum_{n=0}^{n}\left(\frac{1}{3}\right)^n$$

$$=5\frac{1}{(1-\frac{1}{2})}-\frac{1}{1-\frac{1}{3}}=5\frac{1}{\frac{1}{2}}-\frac{1}{\frac{2}{3}}=5(2)-\frac{3}{2}=10-\frac{3}{2}=\boxed{\frac{17}{2}}$$

② Use partial fractions to find the sum of the series $\sum_{n=1}^{\infty}\frac{6}{(2n-1)(2n+1)}$

$$\sum_{n=1}^{\infty}\frac{6}{(2n-1)(2n+1)}=6\sum_{n=1}^{\infty}\frac{1}{(2n-1)(2n+1)}$$

We want to find A and B such that

$$\frac{1}{(2n-1)(2n+1)}=\frac{A}{2n-1}+\frac{B}{2n+1}$$

multiplying this by $(2n-1)(2n+1)$ we got

$$1=A(2n+1)+B(2n-1)=(2A+2B)n+(A-B)$$

that is, $0\cdot n+1=(2A+2B)n+(A-B)$. Thus

$$\begin{matrix}2A+2B=0\\ A-B=1\end{matrix}\Rightarrow\begin{cases}A+B=0\\ A-B=1\end{cases}\Rightarrow\begin{cases}A=-B\\ A-B=1\end{cases}\Rightarrow -2B=1\Rightarrow B=-\tfrac{1}{2}$$

$$\Rightarrow A=-B=\tfrac{1}{2}$$

Then

$$\sum_{n=1}^{\infty}\frac{6}{(2n-1)(2n+1)}=6\sum_{n=1}^{\infty}\left(\frac{1/2}{2n-1}-\frac{1/2}{2n+1}\right)$$

$$=6\sum_{n=1}^{\infty}\left[\frac{1}{4}\left(\frac{1}{n-1/2}\right)-\frac{1}{4}\left(\frac{1}{n+1/2}\right)\right]$$

$$=\frac{6}{4}\left[\sum_{n=1}^{\infty}\frac{1}{n-1/2}-\frac{1}{n+1/2}\right]=\frac{3}{2}\left[\sum_{n=1}^{\infty}\frac{1}{n-1/2}-\frac{1}{n+1/2}\right]$$

$$S_k = \sum_{n=1}^{k}\left[\frac{1}{n-\frac{1}{2}} - \frac{1}{n+\frac{1}{2}}\right] = \left(\frac{1}{1-\frac{1}{2}} - \frac{1}{1+\frac{1}{2}}\right) + \left(\frac{1}{2-\frac{1}{2}} - \frac{1}{2+\frac{1}{2}}\right) + \left(\frac{1}{3-\frac{1}{2}} - \frac{1}{3+\frac{1}{2}}\right) + \cdots + \left(\frac{1}{k-\frac{1}{2}} - \frac{1}{k+\frac{1}{2}}\right)$$

$$= \left(\frac{1}{\frac{1}{2}} - \frac{1}{\frac{3}{2}}\right) + \left(\frac{1}{\frac{3}{2}} + \frac{1}{\frac{5}{2}}\right) + \left(\frac{1}{\frac{5}{2}} - \frac{1}{\frac{7}{2}}\right) + \cdots + \left(\frac{1}{\frac{2k-1}{2}} - \frac{1}{\frac{2k+2}{2}}\right)$$

Removing parentheses and canceling adjacent terms of opposite sign collapses the sum to

$$S_k = 2 - \frac{2}{2k+2}$$

Now observe that $\lim_{k\to\infty} S_k = 2 - 0 = 2$. Therefore

$$\sum_{n=1}^{\infty} \frac{6}{(2n-1)(2n+1)} = \frac{3}{2}\cdot\left(\sum_{n=1}^{\infty}\left(\frac{1}{n-\frac{1}{2}} - \frac{1}{n+\frac{1}{2}}\right)\right)$$

$$= \frac{3}{2}\cdot 2 = 3$$