Quiz 1

Which sequences $\{a_n\}$ converge, and which diverge? Find the limit of each convergent sequence.

① $a_n = \left(\frac{n+1}{2n}\right)\left(1-\frac{1}{n}\right)$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \left(\frac{n+1}{2n}\right)\left(1-\frac{1}{n}\right)$$

note that $a_n$ can be written as the product $a_n = b_n \cdot c_n$ where $b_n = \frac{n+1}{2n} = \frac{n}{2n} + \frac{1}{2n} = \frac{1}{2} + \frac{1}{2n}$

and $c_n = 1 - \frac{1}{n}$

also we have:

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \left(\frac{1}{2} + \frac{1}{2n}\right) = \lim_{n\to\infty} \frac{1}{2} + \lim_{n\to\infty} \frac{1}{2n} = \frac{1}{2} + 0 = \frac{1}{2}$$

$$\lim_{n\to\infty} c_n = \lim_{n\to\infty} 1 - \frac{1}{n} = 1 - 0 = 1$$

Therefore $\lim_{n\to\infty} a_n = \frac{1}{2} \cdot 1 = \frac{1}{2}$

② $a_n = \frac{\sin^2 n}{2^n}$

note that $0 \leq \sin^2 n \leq 1$

then

$$0 \leq \sin^2 n \leq \frac{1}{2^n}$$

if $b_n = \frac{1}{2^n}$ $\lim_{n\to\infty} \frac{1}{2^n} = 0$, by Sandwich theorem

we have $\lim_{n\to\infty} \frac{\sin^2 n}{2^n} = 0$