Practice Problems for the Second Midterm
Math 21A

November 22, 2003

Problem 1 Compute the derivative of $f(x) = \frac{1+\sin x}{\cos^2 x}$.

Problem 2 Compute the derivative of $f(x) = \sin x + \cos(x^2) + \tan(x^3)$.

Problem 3 Consider the function $f(x) = x^4 + x^2 + 2$.

(a) Show that the function satisfies the hypothesis of Rolle's Theorem on the interval $[-2, 2]$.

(b) Find all numbers c in $[-2, 2]$ such that $f'(c) = 0$.

Problem 4 Graph the following functions, and show all x-intercepts, y-intercepts, critical points, and asymptotes.

(a) $f(x) = 2x^3 + 3x^2 - 6x$

(b) $f(x) = \frac{1}{2x^2 - x}$

Problem 5 A car begins to break at 20 feet from a traffic light. Its velocity is initially $10 ft/s$ and it comes to a full stop right at the traffic light. Assuming constant deceleration, how long does it take to stop?

Problem 6 For the following functions, find the inflection points and describe the intervals where the function is concave up or concave down.

(a) $f(x) = 3x^5 - 5x^4$

(b) $f(x) = \sin x$

Problem 7 Consider positive numbers x and y such that $2x + y = 5$. What is the maximum possible value of xy?

Problem 8 Let y be a function such that $x^3 + xy^2 + \sin y^2 = \frac{2\sqrt{2}+4+\pi}{4}$. Find the tanget line to the graph at the point $(1, \frac{\sqrt{\pi}}{2})$.

Problem 9 The radius of a sphere is measured with a 10% error. Find the maximum absolute error on

(a) the surface area of the sphere (Hint: Surface area $= 4\pi r^2$.)

(b) the volume of the sphere (Hint: Volume $= \frac{4}{3}\pi r^3$.)

Problem 10 The length of a leg of a right triangle is increasing at the rate of $5ft/s$, and the length of the other leg is decreasing at the rate of $6ft/s$. What is the rate of change of the hypotenuse when the first leg is 3 feet long and the other leg is 4 feet long? Is the hypotenuse getting longer or shorter at that moment?