Intr.: Ernest Woei *September 1, 2006*

Last name: Solutions *First name:*

PLEASE READ THIS BEFORE YOU DO ANYTHING ELSE!

1. Make sure that your exam contains 7 pages, including this one.
2. **NO** calculators, books, notes or other written material allowed.
3. Express all numbers in exact arithmetic, i.e., no decimal approximations.
4. Read the statement below and sign your name.

   I affirm that I neither will give nor receive unauthorized assistance on this examination. All the work that appears on the following pages is entirely my own.

Signature: ______________________________

"Anyone who has never made a mistake has never tried anything new." –Albert Einstein.

GOOD LUCK!!!

1. Find the indefinite integrals.

(a) (*5 pts*)

$$\int \frac{3\sec^2 u}{8\tan u + 4}\, du$$

derivative of $\tan u$ is $\sec^2 u$

let $v = 8\tan u + 4$

$dv = 8\sec^2 u \, du$

$\frac{3}{8} dv = 3\sec^2 u \, du$

$$\int \frac{3\sec^2 u}{8\tan u + 4}\, du = \int \frac{\frac{3}{8}\, dv}{v} = \frac{3}{8}\int \frac{1}{v}\, dv = \frac{3}{8}\ln|v| + C$$

$$= \frac{3}{8}\ln|8\tan u + 4| + C$$

(b) (*4 pts*)

$$\int 3e^{\sin x^3}\cos(x^3)x^2\, dx$$

derivative of $\sin(x^3)$ is $\cos(x^3)\, 3x^2$

let $u = \sin(x^3)$

$du = \cos(x^3)\, 3x^2\, dx$

$\|$

$$\int e^u \, du = e^u + C$$

$$= e^{\sin(x^3)} + C$$

(c) (*6 pts*)

$$\int \arctan x \, dx$$

Use IBP

$u = \arctan x \qquad dv = dx$

$du = \frac{1}{1+x^2} dx \qquad v = x$

$$\int \arctan x \, dx = x \arctan x - \int \frac{x}{1+x^2} dx$$

$$= x \arctan x - \frac{1}{2} \ln(1+x^2) + C$$

$$\int \frac{x}{1+x^2} dx = \int \frac{\frac{1}{2}}{u} du = \frac{1}{2} \int \frac{1}{u} du = \frac{1}{2} \ln(1+x^2)$$

let $u = 1+x^2$

$du = 2x \, dx \rightarrow \frac{1}{2} du = x \, dx$

(d) (*7 pts*)

$$\int (x+2)^2(x-1)^5 \, dx$$

Using substitution

let $u = x-1 \qquad du = dx$

$u+1 = x$

$$\int (x+2)^2 (x-1)^5 dx = \int (u+3)^2 u^5 du = \int (u^2+4u+4) u^5 du$$

$$= \int u^7 + 4u^6 + 4u^5 du = \frac{u^8}{8} + \frac{4u^7}{7} + \frac{4u^6}{6} + C$$

$$= \frac{(x-1)^8}{8} + \frac{4(x-1)^7}{7} + \frac{4(x-1)^6}{6} + C$$

//

2. Find the definite integrals.

   (a) (*8 pts*)

$$\int_{-3}^{3} |x-1| + 3x - 4\ dx = \int_{-3}^{3} |x-1|\ dx + \int_{-3}^{3} 3x\ dx - \int_{-3}^{3} 4\ dx$$

since $3x$ is an odd function and we are integrating on interval $[-3,3]$ then $\int_{-3}^{3} 3x\ dx = 0$

$$\int_{-3}^{3} 4\ dx = 4 \cdot 6 = 24$$

So we must find $\int_{-3}^{3} |x-1|\ dx$

Note: $|x-1| = \begin{cases} x-1 & x \geq 1 \\ 1-x & x < 1 \end{cases}$

So $$\int_{-3}^{3} |x-1|\ dx = \int_{-3}^{1} 1-x\ dx + \int_{1}^{3} x-1\ dx$$

$$= x - \frac{x^2}{2}\Big]_{-3}^{1} + \frac{x^2}{2} - x\Big]_{1}^{3}$$

$$= 1 - \frac{1}{2} - \left(-3 - \frac{9}{2}\right) + \frac{9}{2} - 3 - \left(\frac{1}{2} - 1\right)$$

$$= \frac{1}{2} + \frac{15}{2} + \frac{3}{2} + \frac{1}{2} = 10$$

thus $$\int_{-3}^{3} |x-1| + 3x - 4\ dx = 10 - 24 = -14$$ //

(b) (*3 pts*)

$$\int_{-\pi}^{\pi} x^2 \sin x \, dx$$

Note: $x^2 \sin x$ is an odd function, thus

$$\int_{-\pi}^{\pi} x^2 \sin x \, dx = 0.$$

(c) (*7 pts*)

$$\int_{-\pi}^{\pi} x \sin x \, dx$$

$x \sin x$ is an even function

so $\int_{-\pi}^{\pi} x \sin x \, dx = 2 \int_0^{\pi} x \sin x \, dx$

Using IBP

$u = x \qquad dv = \sin x \, dx$

$du = dx \qquad V = -\cos x$

$$2 \int_0^{\pi} x \sin x \, dx = 2\left[-x\cos x\Big|_0^{\pi} + \int_0^{\pi} \cos x \, dx\right] = 2\left[-x \cos x \Big|_0^{\pi} + \sin x \Big|_0^{\pi}\right]$$

$= 2\pi$ //

3. (*7 pts*) Find the area of the region bounded by the graphs: $y = \frac{1}{\sqrt{2x+1}}, y = 0, x = 0$, and $x = 4$.

$\frac{1}{\sqrt{2x+1}} \geq 0$ for $0 \leq x \leq 4$

$$\text{Area} = \int_0^4 (2x+1)^{-1/2}\, dx$$

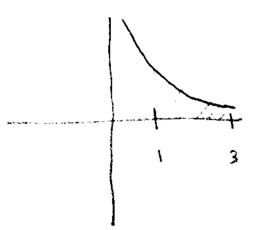
$u = 2x+1$ $\quad$ $x = 0 \to u = 1$

$du = 2\,dx$ $\quad$ $x = 4 \to u = 9$

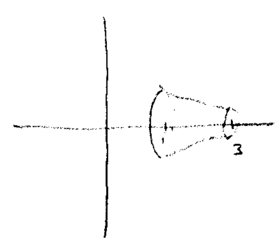
$\frac{1}{2}\, du = dx$

$$= \frac{1}{2}\int_1^9 u^{-1/2}\, du = \frac{1}{2}\left[2u^{1/2}\right]_1^9 = u^{1/2}\Big]_1^9 = 3 - 1 = 2$$

4. (*5 pts*) Find the volume of the solid formed by revolving the region bounded by the graphs of the equations $y = \frac{1}{x}, y = 0, x = 1, x = 3$. about the x-axis



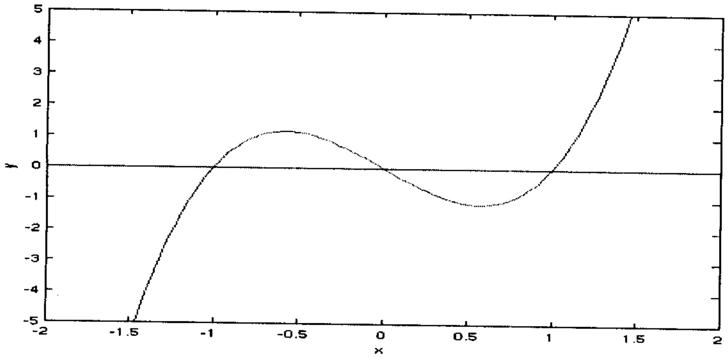
revolve around x-axis



$$\text{Volume} = \pi \int_1^3 \frac{1}{x^2}\, dx = \pi \left.\frac{x^{-1}}{-1}\right]_1^3 = -\pi\left[\frac{1}{x}\right]_1^3$$

$$= -\pi\left[\frac{1}{3} - 1\right] = \frac{2\pi}{3}$$

5. (*8 pts*) Find the area of the region bounded by the graphs of $f(x) = 3(x^3 - x)$ and $g(x) = 0$. The graph of $f(x)$ and $g(x)$ is shown.



$f(x) = g(x) \implies 3(x^3 - x) = 3x(x^2-1) = 3x(x-1)(x+1) = 0$

$\implies x = 0, 1, -1$

$$\text{Area} = \int_{-1}^{0} f(x) - g(x)\, dx + \int_{0}^{1} g(x) - f(x)\, dx$$

$$= 3\left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^{0} - 3\left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{0}^{1}$$

$$= -3\left(\frac{1}{4} - \frac{1}{2}\right) - 3\left(\frac{1}{4} - \frac{1}{2}\right)$$

$$= \frac{3}{4} + \frac{3}{4} = \frac{6}{4} = 3/2 \quad //$$

Page	2 (*9*)	3 (*13*)	4 (*8*)	5 (*10*)	6 (*12*)	7 (*8*)	Total (*60*)
Scores							