Last name: Key *First name:*

PLEASE READ THIS BEFORE YOU DO ANYTHING ELSE!

1. Make sure that your exam contains 6 pages, including this one.
2. **NO** calculators, books, notes or other written material allowed.
3. Express all numbers in exact arithmetic, i.e., no decimal approximations.
4. Read the statement below and sign your name.

   I affirm that I neither will give nor receive unauthorized assistance on this examination. All the work that appears on the following pages is entirely my own.

Signature: ____________________________

"You can profit from your mistakes, but that does not mean the more mistakes, the more profit." - Anonymous

GOOD LUCK!!!

1. (*10 pts*) Suppose you want to deposit \$10,000 into a savings account and leave it in there for 25 years. If the interest is compounded monthly, what interest rate do you need to get to have a balance of \$100,000 after 25 years. Hint: $10^{1/300} \approx 1.008$.

$$B = P\left(1+\frac{r}{n}\right)^{nt} \qquad P = 10^4 \qquad B = 10^5$$

$$n = 12$$

$$t = 25$$

$$10^5 = 10^4\left(1+\frac{r}{12}\right)^{12\cdot 25} = 10^4\left(1+\frac{r}{12}\right)^{300}$$

$$\Rightarrow \quad 10 = \left(1+\frac{r}{12}\right)^{300}$$

$$\Rightarrow \quad 10^{\frac{1}{300}} - 1 = \frac{r}{12}$$

$$\Rightarrow \quad 12\left(10^{\frac{1}{300}} - 1\right) = r$$

$$\Rightarrow \quad 12\left(1.008 - 1\right) \simeq r$$

$$.096 = 12\,(.008) \simeq r$$

You need to get an interest rate of 9.6%. //

2. (*21 pts*) Find the derivative of the following functions:

(a) (*9 pts*) $y = \ln\left(\frac{x^2\sqrt[3]{x^4+x^2}}{e^x}\right) = \ln\left(x^2\sqrt[3]{x^4+x^2}\right) - \ln\left(e^x\right)$

$$= \ln(x^2) + \ln\left((x^4+x^2)^{1/3}\right) - x$$

$$= 2\ln(x) + \frac{1}{3}\ln\left(x^4+x^2\right) - x$$

$$\frac{dy}{dx} = \frac{2}{x} + \frac{1}{3}\,\frac{1}{x^4+x^2}\,4x^3+2x - 1$$

//

(b) (*3 pts*) $y = 4^{x^2+e^x}$

$$\frac{dy}{dx} = \ln 4 \cdot 4^{x^2+e^x} \cdot (2x+e^x)$$

//

(c) (*4 pts*) $y = \left[e^{(4x^3+2x^2+1)}e^{(-2x^2-1)}\right]^{\frac{1}{4}}$ $= \left[e^{4x^3+\cancel{2x^2}+\cancel{1}-\cancel{2x^2}-\cancel{1}} \right]^{1/4}$

$$= \left[e^{4x^3} \right]^{1/4} = e^{x^3}$$

$$\frac{dy}{dx} = 3x^2 \, e^{x^3}$$

//

(d) (*5 pts*) $y = x^x$

$$\ln y = \ln(x^x) = x \ln x$$

$$\frac{d}{dx} \ln y = \frac{d}{dx} x \ln x$$

$$\frac{y'}{y} = \ln x + x \frac{1}{x} = \ln x + 1$$

$$y' = (\ln x + 1) y = (\ln(x) + 1) x^x$$

//

3. (*9 pts*) Let

$$e^{x^2}y + x^2 \ln y = 0$$

Find $\frac{dy}{dx}$. You do not need to simplify.

$$\frac{d}{dx}\left(e^{x^2}y + x^2 \ln y\right) = \frac{d}{dx} 0$$

$$2x e^{x^2} y + \frac{dy}{dx} e^{x^2} + 2x \ln y + x^2 \frac{1}{y}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}\left(e^{x^2} + \frac{x^2}{y}\right) = -2x e^{x^2} y - 2x \ln y$$

$$\frac{dy}{dx} = \frac{-2x e^{x^2} y - 2x \ln y}{e^{x^2} + \frac{x^2}{y}}$$

//

4. (*9 pts*) Find $F(x)$ when

$$F''(x) = x^2 - 2x + 2$$
$$F'(0) = 1$$
$$F(0) = 2$$

$$F'(x) = \int F''(x)\,dx = \int x^2 - 2x + 2 \; dx$$

$$= \frac{x^3}{3} - x^2 + 2x + C$$

$$1 = F'(0) = C \implies \boxed{C = 1}$$

$$F'(x) = \frac{x^3}{3} - x^2 + 2x + 1$$

$$F(x) = \int F'(x)\,dx = \int \frac{x^3}{3} - x^2 + 2x + 1 \; dx$$

$$= \frac{x^4}{12} - \frac{x^3}{3} + x^2 + x + C$$

$$2 = F(0) = C \implies C = 2$$

$$F(x) = \frac{x^4}{12} - \frac{x^3}{3} + x^2 + x + 2$$

//

5. (*18 pts*) Find the indefinite integrals.

(a) (*5 pts*)

$$\int \frac{t^8 + t^4 - t^{3/2}}{t^{3/2}}\, dt$$

$$\int \frac{t^8}{t^{3/2}} + \frac{t^4}{t^{3/2}} - \frac{t^{3/2}}{t^{3/2}}\, dt = \int t^{13/2} + t^{5/2} - 1\, dt$$

$$= \frac{2t^{15/2}}{15} + \frac{2t^{7/2}}{7} - t + C \quad //$$

(b) (*6 pts*)

$$\int \frac{3x^2}{(x^3-1)^2}\, dx$$

$u = x^3 - 1$

$du = 3x^2\, dx$

$$\int \frac{3x^2}{(x^3-1)^2}\, dx = \int \frac{du}{u^2} = \frac{u^{-1}}{-1} + C = \frac{-1}{x^3-1} + C \quad //$$

(c) (*7 pts*)

$$\int \sqrt{x}(4 - x^{3/2})^2\, dx$$

$u = 4 - x^{3/2}$

$du = -\frac{3}{2} x^{1/2}\, dx$

$-\frac{2}{3}\, du = x^{1/2}\, dx$

$$\int \sqrt{x}\,(4 - x^{3/2})^2\, dx = \int u^2 \left(\frac{-2}{3}\right) du = -\frac{2}{3}\int u^2\, du$$

$$= \frac{-2}{3}\,\frac{u^3}{3} + C = -\frac{2}{9}(4 - x^{3/2})^3 + C \quad //$$

6. (*13 pts*) Sketch the graph of

$$f(x) = x^2 \ln x$$

where $f'(x) = x(2\ln x + 1)$ and $f''(x) = 2\ln x + 3$. Hint: $\lim_{x\to 0^+} x^2 \ln x = 0$.

Domain of f : $x > 0$ //

Intercepts : $f(x) = 0 = x^2 \ln x \Rightarrow$ $\cancel{x = 0}$ not in domain or $\ln x = 0 \Rightarrow x = 1$

$(1, 0)$ //

$\lim_{x\to\infty} f(x) = \infty$ // $\lim_{x\to 0^+} x^2 \ln x = 0$ // NO asymptotes

Extrema's $f'(x) = 0 \Rightarrow x(2\ln x + 1) = 0 \Rightarrow x = 0$ or

$e^{-1}\left(-\frac{1}{2}\right) = \left(e^{-1/2}\right)^2 \ln\left(e^{-1/2}\right) = f\left(e^{-1/2}\right)$

$x = e^{-1/2} \Leftarrow 2\ln x + 1 = 0$

$f''(e^{-1/2}) = 2\ln(e^{-1/2}) + 3 = -1 + 3 = 2 > 0$

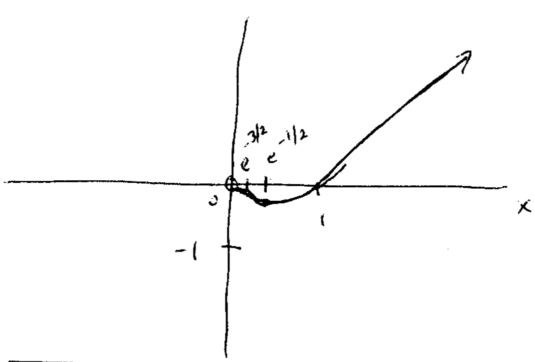
$\Rightarrow$ at $\left(e^{-1/2}, \frac{-1}{2e}\right)$ we have a minima //

Points of inflection : $f''(x) = 0 \Rightarrow 2\ln x + 3 = 0$

$\Rightarrow x = e^{-3/2}$

$f(e^{-3/2}) = e^{-3}\left(\frac{-3}{2}\right) = \frac{-3}{2e^3}$

concave down to concave up at $\left(e^{-3/2}, \frac{-3}{2e^3}\right)$.



$\sim$

$e^{3/2} > e^{1/2}$

$\frac{1}{e^{1/2}} > \frac{1}{e^{3/2}}$

Page	2 (*19*)	3 (*12*)	4 (*18*)	5 (*18*)	5 (*13*)	Total (*80*)
Scores						