Reading a Standard Normal Distribution Table

The *standard normal density probability density function* is

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}.$$

The mean $\mu = 0$ and the standard deviation is $\sigma = 1$. This function does not have an antiderivative that is an elementary function, so we must estimate certain probabilities by using a table.

To compute the probability that x will be less than a,

$$P(\infty < x \leq a) = \int_{\infty}^{a} \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} dx$$

we use the standard normal distribution table.

Example 1

Suppose x is a normal r.v. with mean $\mu = 0$ and standard deviation $\sigma = 1$. Find the probability that $x \leq 1.51$, i.e. $P(\infty < x \leq 1.51)$.

Looking at the table on row 16, column 3 we see the value of this probability will be .9345. Therefore $P(\infty < x \leq 1.51) \approx .9345$.

Example 2

Using the assumptions in Example 1. Find the probability that x will be between 0 and 1.51, i.e. $P(0 \leq x \leq 1.51) = P(x \leq 1.51) - P(x < 0) \approx .9345 - .5 = .4345$.

Example 3

Using the assumptions in Example 1. Find the probability that x will be greater than 1.51, i.e. $P(x > 1.51) = 1 - P(x \leq 1.51) \approx 1 - .9345 = .0655$.

Example 4

This example is from Problem 56 in Section 9.3.

Suppose x is a normal r.v. with mean $\mu = 110$ and standard deviation $\sigma = 10$. Find the probability that x is between 100 and 120, i.e.

$$P(100 \leq x \leq 120) = \int_{100}^{120} \frac{1}{10\sqrt{2\pi}} e^{-\frac{(x-110)^2}{2\cdot 10^2}} dx.$$

We need to convert this integral to an integral using the *standard normal density function.* To do this we will use a substitution. Let $u = \frac{x-110}{10}$, then $du = \frac{1}{10}dx$ so $10du = dx$. We will have to change the limits of integration. For $x = 100$, then $u = \frac{100-110}{10} = -1$. For $x = 120$, then $u = \frac{120-110}{10} = 1$. Thus

$$P(100 \leq x \leq 120) = \int_{100}^{120} \frac{1}{10\sqrt{2\pi}} e^{-\frac{(x-110)^2}{2\cdot 10^2}} dx = \int_{-1}^{1} \frac{1}{10\sqrt{2\pi}} e^{\frac{-u^2}{2}} 10du = \int_{-1}^{1} \frac{1}{\sqrt{2\pi}} e^{\frac{-u^2}{2}} du.$$

Which is exactly in the form of the integral of the *standard normal density function.* So $P(100 \leq x \leq 120) = P(-1 \leq u \leq 1) = P(u \leq 1) - P(u < -1)$. Note that $P(u < -1) = 1 - P(u < 1) \approx 1 - .8413 = .1587$. Thus $P(100 \leq x \leq 120) = P(-1 \leq u \leq 1) = P(u \leq 1) - P(u < -1) \approx .8413 - .1587 = .6826$